Original Article

The Henderson-Hasselbalch Equation: A Three Dimensional Teaching Model

Vinay Oommen, Gnanasenthil Ganesh, Kamalakannan Vadivel and Praghalathan Kanthakumar

Department of Physiology, Christian Medical College, Vellore, Tamil Nadu, India

Abstract

The Henderson-Hasselbalch equation can be considered as the backbone of acid base physiology. This is conventionally represented using two dimensional plots. Although two dimensional plots are simple to use, the equation in reality represents a surface in three dimensional space. Any combination of PaCO₂, [HCO₃⁻] and blood pH values representing acid base disorders is restricted to this surface.

Two models depicting the three dimensional surface generated by the Henderson-Hasselbalch equation were constructed from easily available materials. The first model was constructed using coloured beads, thin metal rods and plywood. This model depicted the Henderson-Hasselbalch surface as a collection of discreet points. The second model depicted the Henderson-Hasselbalch equation as a continuous surface using polystyrene sheets and white cement. The models were presented to undergraduate and post-graduate medical students along with other conventional two dimensional nomograms.

Three dimensional models of the Henderson-Hasselbalch equation can serve as supplementary teaching material to ensure a deeper understanding of acid base physiology.

Introduction

The Henderson-Hasselbalch equation describes the relationship between arterial blood pH, arterial PCO₂ (PaCO₂) and plasma bicarbonate concentrations.

A knowledge of this equation is essential for understanding the processes involved in acid-base balance in the body.

There are many two dimensional plots designed based on the relationship between the three parameters of the bicarbonate-carbonic acid buffer system. The commonly used acid-base nomogram depicts pH on the horizontal axis, [HCO₃⁻] on the vertical axis, and shows PCO₂ as isopleth lines on this 2D surface (1, 2, 3). This two dimensional contour plot of the Henderson Hasselbalch equation is shown in Fig. 1. The Davenport diagram...
incorporates non-bicarbonate titration lines on the acid-base nomogram (4). Siggaard-Andersen and Müller-Plathe have also used 2D nomograms to describe acid base disorders (1, 5).

Plotting the Henderson-Hasselbalch equation, using the variables pH, PaCO₂ and [HCO₃⁻], generates a surface in a three dimensional space. However for reasons of simplicity, the nomograms that are widely used, depict this surface on a two dimensional plane. This hides one of the dimensions and renders the three dimensional surface plot into a two dimensional contour plot (Fig. 1). The axis that is not shown on the graph is represented using isopleth lines. Although the isopleth lines obviate the necessity for an extra dimension, it undoubtedly takes away the intuitive understanding that a three dimensional graph would impart. Two dimensional graphs are easy to draw and reproduce on a paper or a blackboard. Therefore, these are routinely used for teaching acid-base balance during physiology lectures.

In this article we suggest models through which the Henderson-Hasselbalch equation can be represented in its original three dimensional form. These models represent all three variables of the Henderson-Hasselbalch equation on separate axes. These models can be used in the class room during a lecture on acid-base disorders to supplement the standard acid-base nomograms. This helps students visualize the real surface of the equation thereby enhancing their understanding of acid-base regulation.

Methods

**Materials required**

Model 1: Colored beads, thin metal rods (bicycle spokes) and plywood.

Model 2: Polystyrene sheets, instant glue, white cement/gypsum plaster (plaster of Paris) and paints.

**Construction of the model**

**Model 1:**

This model depicts the Henderson-Hasselbalch surface as a collection of discreet points marked by colored beads.

A wooden base, 30 cm x 30 cm was made from plywood. The pH and pCO₂ axes were marked on the wooden base. Fine holes were drilled on the base such that they could firmly hold a thin metal rod when inserted into it. Colored beads were attached to one end of these thin metal rods (Fig. 2A). The metal rods were then cut at different lengths according to the HCO₃⁻ values, such that the height of these metal rods represented plasma HCO₃⁻ concentration on the vertical axis. HCO₃⁻ concentrations were calculated for different sets of pH and pCO₂ values using a spreadsheet and a rearranged version of the Henderson-Hasselbalch equation shown below.

\[ \text{[HCO}_3^-\text{]} = 0.0301 \times \text{PaCO}_2 \times 10^{(\text{pH}-6.1)} \]

The pH axis ranged from 7.0 to 7.7 at 0.05 increments. The PCO₂ axis ranged from 8 to 120 mmHg at increments of 8 mmHg. And, the bicarbonate axis had a range between 0 and 60 mEq/L. The points that had HCO₃⁻ values more than 60 mEq/L were omitted. (The commonly used 2D acid-base nomogram uses a HCO₃⁻ range between 0 mEq/L and 60 mEq/L. Omitting the points larger than 60 mEq/L helped the model better resemble the 2D nomogram. It also served to limit the height of the model).

The beads represented the points that lie on the surface created by the Henderson-Hasselbalch equation. Each of the PCO₂ lines were coded using different colored beads. This enables easy visualization of the PCO₂ isopleth lines. The final assembly of the model is shown in Fig. 2B.

**Model 2:**

This model depicted the Henderson-Hasselbalch
Fig. 1: A two dimensional contour plot representing the Henderson-Hasselbalch equation showing pH on the x-axis, [HCO$_3^-$] on the y-axis and PCO$_2$ as isopleth lines.

Fig. 2. A: Schematic diagram of the materials used. A plywood base board on which the pH (7.0–7.7, interval 0.05) and PCO$_2$ (8-120 mmHg, interval 8 mmHg) axes are represented. Fine holes are drilled in the plank at regular intervals. Thin metal rods, with beads at one end, cut to the appropriate length, represent [HCO$_3^-$]. B: The assembled model showing the surface generated by the Henderson-Hasselbalch equation.
equation as a continuous surface. This surface can be used for writing on while teaching.

The linear relationship between \([\text{HCO}_3^-]\) and \(\text{PCO}_2\) was exploited to ensure the accuracy of the various points of the surface.

Quadrangular pieces were cut from polystyrene sheets. The thickness of each piece corresponded to the scaled distance between 0.05 pH points. Fifteen such pieces were made to represent the pH range of 7.0 to 7.7. The length of the base represented the range of the \(\text{PCO}_2\) axis (from 8 mmHg to 120 mmHg). The height of each piece represented the \(\text{HCO}_3^-\) axis. For each piece the \(\text{HCO}_3^-\) value at a \(\text{PCO}_2\) of 8 was calculated and this was represented by the height at one end. The height of the other end represented the \(\text{HCO}_3^-\) values at a \(\text{PCO}_2\) of 120. As the relationship between \(\text{HCO}_3^-\) and \(\text{PCO}_2\) is linear, a straight line drawn between these two points accurately represents the \(\text{HCO}_3^-\) points in between for that pH value. The last seven pieces were truncated at a constant height. In this model the height corresponded to the \(\text{HCO}_3^-\) value of 70 mEq/L.

The fifteen pieces of polystyrene were then stuck together as shown in Fig. 3A. This provided a step-like three dimensional structure. The gaps between these steps were filled with white cement (gypsum plaster or plaster of Paris can also be used). This created a smooth surface representing the Henderson-Hasselbalch equation. The surface was then painted over so that it could be written on with a marker pen during teaching. The completed model with a few reference markings is shown in Fig. 3B.

**Presentation of the models:**

The models were presented during didactic lectures on acid-base balance to the medical students as well as the physiology graduate students of Christian Medical College, Vellore. These models were used to explain various types of acid-base disorders. It was explained to the students how the point of normality moves to different regions along the surface generated by the Henderson-Hasselbalch equation during acid-base disorders. Figure 4 shows the various regions of acid-base imbalances marked on one of the 3D models. The compensatory responses were also explained. The fact that, the Henderson-Hasselbalch equation restricts freedom of movement to a small surface on a vast three dimensional space was largely appreciated by the students.

![Fig. 3](image_url)

**Fig. 3:** A: Schematic diagram of the materials used. Quadrangular pieces of polystyrene sheets are stuck together. The curved lines represent the areas to be filled to obtain a smooth surface. The inset shows a quadrangular piece of polystyrene for a single pH. The length of the base represents the range of the \(\text{PCO}_2\) axis. \(h_1\) represents the \(\text{HCO}_3^-\) value at a \(\text{PCO}_2\) of 8 mmHg. \(h_2\) represents the \(\text{HCO}_3^-\) value at a \(\text{PCO}_2\) of 120 mmHg. B: Final model after the surface is created and painted.
This can be used to mark the regions of various acid-base disorders.

A two dimensional contour plot of the Henderson-Hasselbalch equation though easy to use, is in reality a simplification of the true three dimensional surface. This surface is brought out in the models we describe and enables the viewer to develop a richer understanding of the topic and the rationale behind the construction of the two dimensional plot.

The inter relationship of all three parameters that contribute to acid base balance is well demonstrated using these models. It is easy to understand how and why a change in one parameter results in a change in the others.

The first model described here can also be constructed using other materials such as thin wooden sticks or drinking straws. In our model metal rods and beads were used for durability. Computer programs that can plot 3D surfaces can also be used for this purpose. However this would require propriety software and the plot would eventually be viewed on a two dimensional display or a projection screen. The true three dimensional nature of the surface is only appreciated while the image is rotating.

Conclusion

A three dimensional representation of the Henderson-Hasselbalch equation can be used along with conventional 2D acid-base nomograms during physiology teaching to enhance the understanding of students. This can be made as a physical model or a computer-generated 3D graph. In this article we have described the making of the physical model of the Henderson-Hasselbalch equation in two different ways. These can be easily made using easily available and inexpensive materials.
References


